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Cahn, Harold Mayer; Atwater, Vern Edward; Cahn, Harold Mayer; Atwater, Vern Edward

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HAROLD MAYER CAHN AND
VERN EDWARD ATWATER

48

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By

Lieut. Harold Mayer Cahn, U. S. Navy (CEC)

and

Lieut. (j.g.) Vern Edward Atwater, U. S. Navy (CEC)

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Preface

In this thesis an attempt was made to measure the reactions at the supports of a thin plate subjected to hydrostatic pressure and simply supported at three edges. Springs were used as a method of measurement. The authors concluded that this method was unsatisfactory.

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Object and Definition

The object of this thesis is to determine the reactions of a rectangular thin plate simply supported along three edges and subjected to hydrostatic pressure; and to compare the results with those obtained by the theoretical analysis of caissons now used by the Bureau of Yards and Docks.

In reality, the caisson slab is of cellular construction; that is, it consists of an exterior skin plating and a series of interior bulkheads or framing. But its general behavior in deforming under hydrostatic loading is similar to that of a thin slab with solid webbing.

For those who are unfamiliar with a dry dock, it may be well to point out the operation of the caisson. When it is desired to remove water from the dock, the caisson is floated into position at the open end of the dock. The ballast tanks are then filled until the caisson rests against its seat. As water is pumped from the dock, the pressure from without forces the caisson more firmly against the seat. After all water has been removed, the caisson becomes, in effect, the flat plate supported on three sides and subjected to hydrostatic pressure, as previously mentioned.

Scope

The complete analysis of the caisson theory would include a study of deflections throughout the plate as well as the reactions at the supports. This thesis is confined to an investigation of the reactions. A thesis by Messrs. G. E. Livingston and W. F. Lorenz at this Institute in August 1948 considered the question of the deflections.

History and Importance

Prior to World War II, a new analysis of dry dock caissons was introduced by Mr. A. Amirikian, Principal Engineer of the Bureau of Yards and Docks. Substantial savings were thereby realized in the cost of caissons constructed during the war. This analysis was based on the deflection expression for a slab of uniform thickness, simply supported on three sides and subjected to hydrostatic loading. The expression may be written as: (1)

$$Z = \frac{5a_o x}{4}\left(\frac{5b^4}{4} - 6b^2y^2 + 4y^4\right) + \sum \cos\frac{n\pi y}{b}\left[A_n \cos h\frac{n\pi x}{b} + B_n \sin h\frac{n\pi x}{b} + C_n x \sin h\frac{n\pi x}{b} + D_n x \cos h\frac{n\pi x}{b}\right] \qquad \text{------ (1)}$$

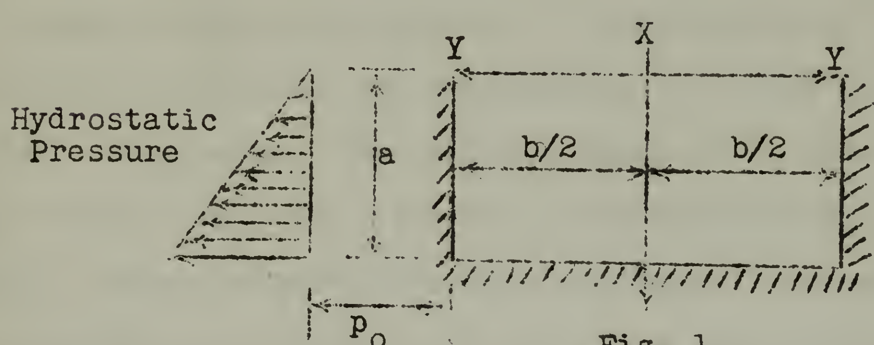

Fig. 1

in which, by reference to Fig. 1,

$$a_o = \frac{p_o}{120\ N_a} \qquad \text{--------(2)}$$

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(1) From notes by Mr. A. Amirikian

A, B, C, and D are constants whose values are determined from the following boundary conditions of the plate or caisson:

(a) No deflection at the three supports.

(b) No moment along the three supports.

(c) Transverse moment at the unsupported edge is zero.

(d) Reaction at the unsupported edge is zero.

In equation form, the boundary conditions are:

$$\left.\begin{aligned}
&Z = 0 \text{ at } x = a, \quad y = \pm b/2 \\
&\Delta z = \frac{d^2 z}{dx^2} + \frac{d^2 z}{dy^2} = 0 \text{ at } x = 0 \qquad y = \pm b/2 \\
&Mx = \frac{d^2 z}{dx^2} + \mu \frac{d^2 z}{dy^2} = 0 \text{ at } x = 0 \\
&R_x = \frac{d^3 z}{dx^3} + (2 - \mu) \frac{d^3 z}{dx\, dy^2} = 0 \text{ at } x = 0
\end{aligned}\right) \; 3$$

By substituting the value of Z from Equation (1) in Equation (3), values of the constants A, B, C and D are obtained. In general, two or three terms of n, that is $n = 1$, 3 and 5 are sufficient for a reasonably accurate solution. After determining these constants, the moment, shear and other expressions in connection with bending of the caisson as a whole are obtained. The equations for the reactions at the supports are:

$$R_x = \frac{d^3 z}{dx^3} + (2 - \mu) \frac{d^3 z}{dx\, dy^2}$$

$$= -\cos \frac{n \pi y}{b} (1 - \mu) \frac{n^3 \pi^3}{b^3} \Big[A_n \sin h \frac{n \pi x}{b}$$

$$+ B_n \cos h \frac{n \pi x}{b} + (x \cos h \frac{n \pi x}{b}$$

$$+ \frac{\mu + 1}{\mu - 1} \frac{b}{n \pi} \sin h \frac{n \pi x}{b}) C_n$$

$$+ \left(x \sin h \frac{n\pi x}{b} + \frac{\mu + 1}{\mu - 1} \frac{b}{n\pi} \cos h \frac{n\pi x}{b}\right) D_n \Bigg]$$

$$-(2 - \mu)\, 15a_o\, (b^2 - 4y^2)$$

$$R_y = \frac{d^3 z}{dy^3} + (2 - \mu) \frac{d^3 z}{dx^2 dy}$$

$$= - \sin \frac{n\pi y}{b} (1 - \mu) \frac{n^3 \pi^3}{b^3} \Bigg[A_n \cosh h \frac{n\pi x}{b}$$

$$+ B_n \sin h \frac{n\pi x}{b} + (x \sin h \frac{n\pi x}{b}$$

$$+ \frac{2 - \mu}{1 - \mu} \frac{2b}{n\pi} \cos h \frac{n\pi x}{b}) C_n$$

$$+ \left(x \cos h \frac{n\pi x}{b} + \frac{2-\mu}{1-\mu} \frac{2b}{n\pi} \sin h \frac{n\pi x}{b}\right) D_n \Bigg]$$

$$+ 60a_o\, bx.$$

For solution of these equations as applied to this thesis, see appendix.

Although the analysis assumes that there is no deflection at the supports, it is well known that the lower corners of actual caissons pull away from their seats. This fact is confirmed by the small amount of leakage observed at the lower corners. However, it is believed that this inconsistency between the actual and theoretical does not materially affect the design. It is handled in the design by introducing imaginary concentrated forces at the lower corners to eliminate the deflection there. By taking experimental data both with the corners held, as assumed by theory, and with the corners free, as is actually the case, information would be made available to the Bureau which would either prove the accuracy of the present analysis or point out where changes should be made.

Discussion

The authors were confronted with the problem of designing a structure which would have this essential feature: the supports must be rigid; but they must also furnish some means of measuring the reactions at enough points to establish the pattern. Of the possible answers to the problem, those listed below were considered to have the most merit. However, those characteristics of the test structure which were fixed in the original planning should be stated first. A commonly designed caisson is one that has a ratio of length to height of three to one. It was therefore decided to build a structure to accomodate a test plate thirty-six inches long by twelve inches high. It was also determined that the reactions should be measured at one-inch intervals. Aluminum was chosen as a material that could be easily fabricated into various shapes. Finally plates of various thickness were to be used, beginning with 14 gauge.

Now, the methods of attacking the problem were:

(a) Plate supported by studs and loads measured by change in electrical resistance.

This method was discarded because the low magnitude of the load which would come to each stud would make accurate measurements difficult. The total force to be measured is,

Area x pressure at C. G.

$= 3$ ft x 1 ft x 1/2 ft x 62.4 lbs. per cu.ft.

$= 93.6$ lbs.

If this force were evenly distributed there would be a load of 1.6 pounds at each of the fifty-eight points

of measurement. By studying the theoretical curve, it was estimated that the maximum load would be about two times the average value, or about 3.2 pounds. This was not considered sufficient to produce the required results. The use of mercury instead of water for the pressure would have given larger loads, but the difficulty in obtaining and handling the necessary one hundred pounds of mercury made this impractical.

(b) Plate supported by quartz crystals through which high-frequency currents could be passed. This offered attractive possibilities, but it was felt a simpler and more inexpensive method should be tried first.

(c) Plate supported by aluminum strips on which electric strain gages might be mounted. Again, the small magnitude of the loads became an important factor. It was found that the loads could be amplified by a beam arrangement and an accurate determination of a small load obtained. This was set aside because of the expense involved and the limited time available. It would have been necessary to machine each part and the number of parts involved would have been too great.

(d) Plate supported by springs.

Springs provided the simplest means of force measurement, but they increased the difficulty of satisfying the requirements of rigid supports. However, the authors believed that this difficulty could be overcome by maintaining the vertical alignment of the test plate by

adjusting the springs as explained later. Therefore, springs were adopted as the means of support.

Another problem to be faced was a means of confining the water. The water had to be prevented from flowing around the edges of the plate, but the seal used could not take any substantial amount of force that should be transmitted to the plate. Professor Grant K. Palsgrove of Rensselear Polytechnic Institute suggested that the liquid might be held in a flexible bag. This suggestion was followed, and this particular feature of the design appeared to be successful.

Description of Test Structure

Essentially the test structure consisted of a plastic bag filled with water and sandwiched between two metal plates. One plate (A) was attached to the inner side of a box by means of eight channel shapes (B). The other plate (C) was suspended from two supports (D) by means of wire. The plastic bag was suspended from a rod (E) that rested on these same supports. Lateral movement of plate (C), which was the test plate representing the dry dock caisson, took place against the action of fifty-eight extension springs spaced at one-inch intervals along the two sides and the bottom of the plate and one-half inch from the edges. The springs were held by means of one-eighth inch bolts at either end. The bolts were threaded through holes in the test plate and in the side of the box. Thus, the load at each point of measurement was transmitted from the plate by a bolt, from the bolt to a spring, thence to another bolt, and finally to the side of the box. It was decided that no provision would be made to hold the lower corners until action of the plate with corners free was observed.

Since the load on each spring was expected to vary from zero to about three and one-half pounds, a spring with a rating of two pounds per inch was desirable. The springs were designed by the formula,

$$e = \frac{8N D^3 P}{d^4 F}$$

e = deflection = 1 inch

N = number of coils

d = diameter of wire; a convenient size that was available was 0.040 inch.

P = load in lbs = 2 lbs.

F = shearing modulus of elasticity = 12,000,000 lbs. per sq. in.

D = mean diameter of spring; the outside diameter was limited to one-half inch by the clearance between measurement points, giving a D of 0.460 inch with .040 inch wire.

Solution of the formula with values as stated above gave an N of 20 turns.

A sample spring was made according to the above specifications, and it appeared satisfactory. Therefore, the required fifty-eight were manufactured.

Each spring was calibrated by measuring the length of the spring with various weights hanging on it. Most springs were measured at zero weight, one pound and four pounds. However, several were checked at one-half, two, three and five pounds. The deflections per pound were computed by taking one-fourth of the deflection at four pounds. Values obtained are listed on next page.

Spring No.	Zero Length	In.Defl. per lb.	Spring No.	Zero Length	In.Defl. per lb.
1	1.78	0.445	31	1.76	.488
2	1.81	.465	32	1.76	.500
3	1.80	.572	33	1.80	.550
4	1.76	.550	34	1.76	.440
5	1.74	.455	35	1.80	.532
6	1.81	.582	36	1.77	.542
7	1.81	.508	37	1.80	.532
8	1.84	.535	38	1.85	.490
9	1.86	.550	39	1.76	.450
10	1.82	.450	40	1.73	.452
11	1.84	.540	41	1.75	.462
12	1.79	.440	42	1.77	.442
13	1.79	.442	43	1.83	.538
14	1.83	.540	44	1.85	.542
15	1.82	.538	45	1.81	.522
16	1.81	.528	46	1.78	.448
17	1.80	.452	47	1.78	.448
18	1.79	.430	48	1.83	.552
19	1.85	.512	49	1.76	.455
20	1.78	.450	50	1.81	.485
21	1.83	.552	51	1.75	.488
22	1.79	.430	52	1.76	.455
23	1.82	.458	53	1.76	.438
24	1.77	.440	54	1.84	.510
25	1.79	.440	55	1.75	.455
26	1.79	.445	56	1.77	.450
27	1.80	.528	57	1.83	.528
28	1.78	.448	58	1.82	.538
29	1.76	.442			
30	1.75	.445			

Experimental Procedure

The success of the experiment depended on the possibility of maintaining the three supported edges of the test plate in one vertical plane after the load was applied. This was to be done by adjusting the springs by means of the bolts that held the springs to the plate and to the box.

A transit was used to align the plate. The transit was set up about fifteen feet from one edge of the plate, which was suspended one inch from the back-up plate (A), and a line of sight was established to this edge. The vertical cross-hair was then used to put the edge in a vertical plane. Then the entire structure was moved until the far edge was placed in the same plane. This established a reference plane, and it was intended that the plate would be returned to this identical position after loading.

The next step was to attach the springs. The springs were made so that the diameter of the last few turns on either end were less than the diameter of the center turns. In other words, the springs were coned down on both ends. These narrowed ends were the means for holding the heads of the one-eighth inch bolts. One bolt on each spring was screwed into the test plate, while the other bolt was screwed into the opposite hole in the box. Each bolt was taken up about one inch.

The test plate was now ready for loading. Water was poured into the plastic bag until it was level with the top of the test plate. The bag was moved slightly so that

it rested firmly against the test plate and so that the pressure of the water acted against the entire area of the plate within the springs.

The plate now assumed this position: The side edges sloped so that the bottom corners were one-half inch from the back-up plate while the top corners were about two inches away. The bottom edge bowed outward; that is, the center portion was about two and one-half inches from the back-up plate.

At this point the authors had planned to adjust the tension in each spring, by turning the bolts attached to the spring, until the three edges of the plate were in the previously established plane. For example, if the plate were out of alignment at a point adjacent to a particular spring, the tension on that spring would be changed until the point was back on the vertical cross-hair of the transit. However, this procedure was prevented by the stiffness of the plate. When a spring was moved, an entire side of the plate moved; and there was no local deflection as had been hoped. Thus, there were an infinite number of spring settings that would align the plate. Two such settings were recorded, the average deflections determined, and the forces computed. Between readings, the plate was thrown out of alignment and then readjusted with different spring tensions.

To investigate conditions with the corners held, springs were attached to the lower corners and to the side of the

box opposite to that which held the main springs. These corner springs had to be stiffer because the force was much greater than at any other single point. Therefore a spring with a rating of five pounds per inch and five pounds initial compression was used. The plate was aligned and readings taken in a manner similar to the procedure with the corners not held.

Results

Two sets of spring readings were taken with the corners free and two with the corners held, as explained in the previous section. Since the readings were considered of little value in this problem, only two of each were taken. It should again be emphasized that an infinite number of combinations of spring tensions would have aligned the plate.

Readings taken and forces computed are given on the following pages.

Measurement of Forces, Corners Not Held

(Note: Spring Numbers correspond to numbers of holes on drawing)

Sides:

Right

Spring No.	Length Spring Reading #1	Reading #2	Ave. Defl.	Forces lbs.
1	3.82	3.68	1.97	4.43
2	3.58	3.54	1.75	3.76
3	3.26	3.23	1.44	2.52
4	3.08	3.11	1.28	2.33
5	2.76	2.77	1.02	2.24
6	2.55	2.54	.73	1.25
7	2.73	2.74	.93	1.83
8	--	--	0	0
9	1.98	2.03	.14	.25
10	--	--	0	0
11	--	--	0	0
12	--	--	0	0

Left

58	3.50	3.09	1.48	2.75
57	3.49	2.99	1.41	2.67
56	3.10	3.05	1.31	2.91
55	2.95	2.98	1.21	2.66
54	2.76	3.45	1.26	2.47
53	2.33	2.40	.60	1.37
52	2.26	2.31	.52	1.14
51	2.00	2.22	.36	.74
50	1.98	2.14	.12	.25
49	--	1.85	.04	.09
48	--	--	0	0
47	--	--	0	0

Bottom: (Corners Not Held)

Spring No.	Length Spring Reading #1	Reading #2	Ave. Defl.	Forces lbs.
12	--	--	0	0
13	--	--	0	0
14	--	--	0	0
15	--	--	0	0
16	--	--	0	0
17	--	--	0	0
18	--	--	0	0
19	--	--	0	0
20	1.96	--	.21	.46
21	2.19	--	.18	.33
22	2.40	2.11	.47	1.09
23	2.37	2.26	.50	1.09
24	2.37	2.91	.97	2.20
25	2.68	2.95	1.03	2.34
26	2.60	2.89	.91	2.04
27	3.32	3.46	1.59	3.01
28	3.21	3.42	1.54	3.44
29	3.25	3.40	1.57	3.56
30	3.31	3.39	1.60	3.60
31	3.35	3.32	1.58	3.24
32	2.93	3.32	1.35	2.70
33	2.35	2.59	.67	1.22
34	2.54	2.41	.72	1.64
35	2.48	2.01	.44	.83
36	2.47	2.03	.48	.88
37	2.24	1.99	.32	.60
38	2.08	0	.12	.24
39	2.08	1.78	.17	.38
40	2.00	1.78	.16	.35
41	1.85	--	.05	.11
42	1.87	--	.05	.11
43	1.84	--	.01	.02
44	--	--	0	0
45	--	--	0	0
46	--	--	0	0
47	--	--	0	0

Measurement of Forces, Corners Held

(Note: Spring numbers correspond to numbers of holes on drawing)

Sides:

Right

Spring No.	Length Spring Reading #1	Reading #2	Ave. Defl.	Forces lbs.
1	3.27	3.02	1.36	3.06
2	3.20	3.19	1.39	2.99
3	3.04	3.14	1.29	2.25
4	3.28	3.28	1.52	2.76
5	3.02	3.02	1.28	2.81
6	2.82	2.81	1.01	1.74
7	3.08	3.10	1.28	2.52
8	2.17	2.59	0.57	1.06
9	2.49	2.56	.66	1.20
10	--	--	0	0
11	--	--	0	0
12a	3.92	4.08	4.00*	-6.63

Left

58	2.99	3.02	1.18	2.19
57	2.84	2.86	1.02	1.93
56	2.96	2.98	1.20	2.67
55	3.08	3.09	1.33	2.92
54	3.68	3.70	1.85	3.62
53	2.69	2.50	0.83	1.89
52	2.64	2.43	0.77	1.69
51	2.62	2.30	0.71	1.45
50	2.53	2.19	0.55	1.13
49	--	--	0	0
48	--	--	0	0
47a	4.79	4.25	4.52*	-8.92

Bottom: (Corners Held)

Spring No.	Length Spring Reading #1	Reading #2	Ave. Defl.	Forces lbs.
12a	-3.92	-4.08	4.00*	-6.63
13	--	--	0	0
14	--	--	0	0
15	1.88	1.93	.08	.15
16	--	--	0	0
17	--	--	0	0
18	1.80	--	.01	.02
19	--	--	0	0
20	--	--	0	0
21	1.92	1.92	.09	.16
22	2.20	2.19	.41	.95
23	2.34	2.34	.52	1.13
24	2.92	2.93	1.15	2.62
25	2.95	2.94	1.15	2.62
26	3.18	3.19	1.39	3.12
27	3.43	3.40	1.62	3.07
28	3.40	3.40	1.62	3.62
29	3.25	3.25	1.49	3.38
30	3.38	3.38	1.63	3.66
31	3.38	3.39	1.62	3.32
32	3.37	3.38	1.60	3.20
33	3.38	3.37	1.58	2.87
34	3.44	3.46	1.69	3.84
35	2.98	3.01	1.20	2.26
36	2.60	2.60	.83	1.53
37	2.41	2.03	.42	.79
38	2.18	1.93	.21	.43
39	1.94	0	.09	.20
40	1.89	0	.08	.18
41	1.76	1.77	.01	.02
42	1.78	--	.01	.02
43	--	--	0	0
44	--	--	0	0
45	1.84	--	.01	.02
46	--	--	0	0
47a	4.79	4.25	4.52*	-8.92

* Actual lengths of springs used to hold corners.
Calibration of these springs as follows:

	Zero	5 lb.	12 lb.
12a	3.65	3.65	5.15
47a	3.63	3.63	5.22

Conclusion

The problem presented by this thesis is an intriguing one, and one that does not readily yield to a solution. There are numerous answers, as stated in the "Discussion", that appear to be perfect on the surface, but on further analysis show rather serious defects. The authors believe that springs are not the answer, but that this was the proper method to attempt first because of its comparative inexpensiveness and simplicity.

The results obtained neither proved nor disproved the theoretical analysis. The most valuable contribution would have been a comparison of seat pressures both with the corners held and the corners free. This would have given a comparison between the actual and the theoretical. But not enough reliance may be placed on the results from the use of springs to draw any conclusions.

The pattern obtained with the corners free in no way resembles the theory. This may be blamed on the fact that there was not sufficient flexibility between points of support to determine when a spring was carrying its proper share of the load. Although some other method of approach is recommended, use of a thinner plate might be investigated. The plate must be so flexible that it deflects locally when there is a change of spring tension.

The pattern of support reactions with the corners held bears some resemblance to the theory; that is, the values increase from the top down to some maximum value and then back to zero. At the lower corners there is a concentrated

force in the opposite direction. Also, the reactions rise to a maximum at the bottom. But in this set-up the corner springs were working against the main springs and it was impossible to tell which ones to adjust to bring the plate into alignment. For example, if the tension on one of the lower side springs were increased, the tension on the spring holding the corner would have to be increased to realign the plate. It was impossible to tell what combination of spring settings was correct. Here, again, a thinner plate might have served better, but there would still have been some errors in the lowest of the side readings.

The authors suggest that the best method of conclusively checking the caisson analysis would be to construct a large scale model, say one-fourth to one-third size. Then the seat pressures could be measured with strain-gage technique.

One fact that was established by this thesis was that there is a considerable force required to hold the lower corners to the seat. For the reasons stated, this force could not be accurately measured. Further study is suggested to determine the effect of varying plate thickness and pressure head on the magnitude of this force.

PLATE A

BACK-UP PLATE
24 ST ALUMINUM
SCALE: 1/4" = 1.0"

PLATE B

CHANNEL
24 ST ALUMINUM
SCALE: 1/2" = 1.0"

PLATE C

TEST PLATE
14 GAGE ALUMINUM
SCALE: 9/16" = 1"

PLATE D

HANGER SUPPORT
24 ST ALUMINUM
SCALE: 1/4" = 1"

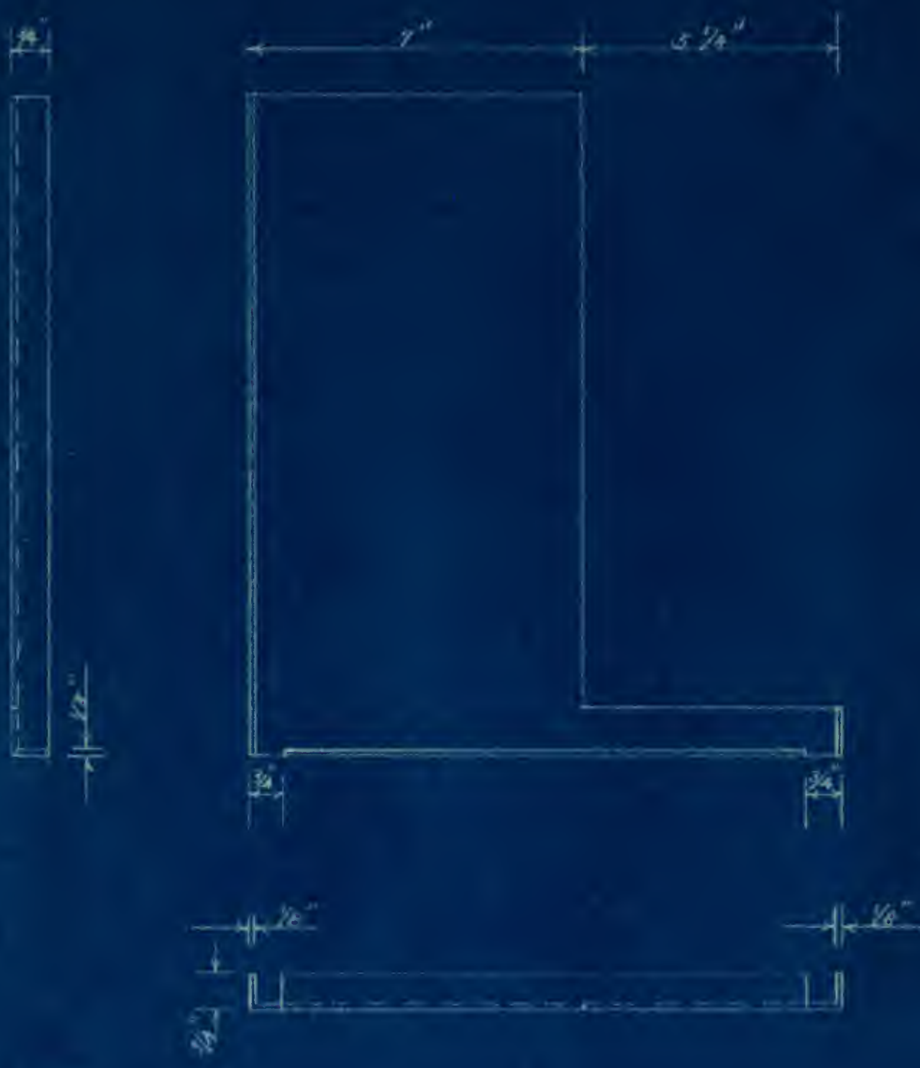
PLATE F

FLUID BAG
PLASTIC
SCALE: 3/16" = 1"

PLATE G

SIDE PLATE
24 ST ALUMINUM
SCALE: 1/4" = 1"

PLATE H



END PLATE
24 ST ALUMINUM
SCALE: 1/4" = 1"

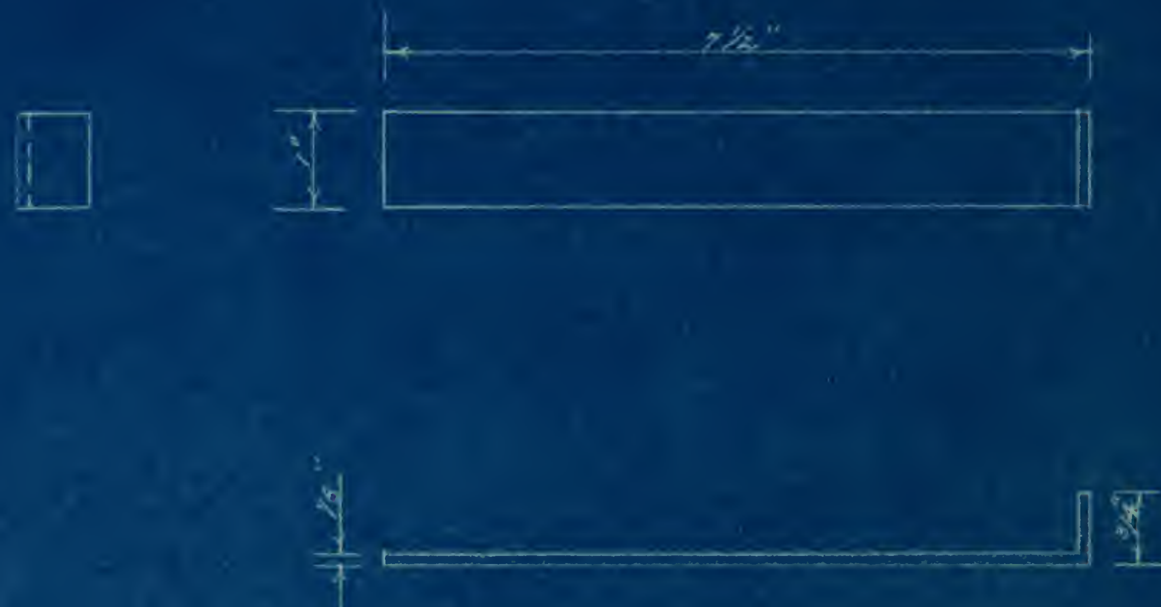
PLATE I

BOTTOM PLATE

24 ST ALUMINUM

SCALE: 3/16" = 1"

PLATE J

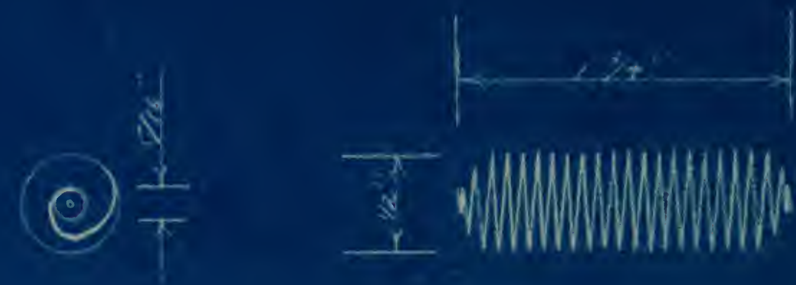


TOP BRACE

24 ST ALUMINUM

SCALE: ½" = 1"

PLATE K



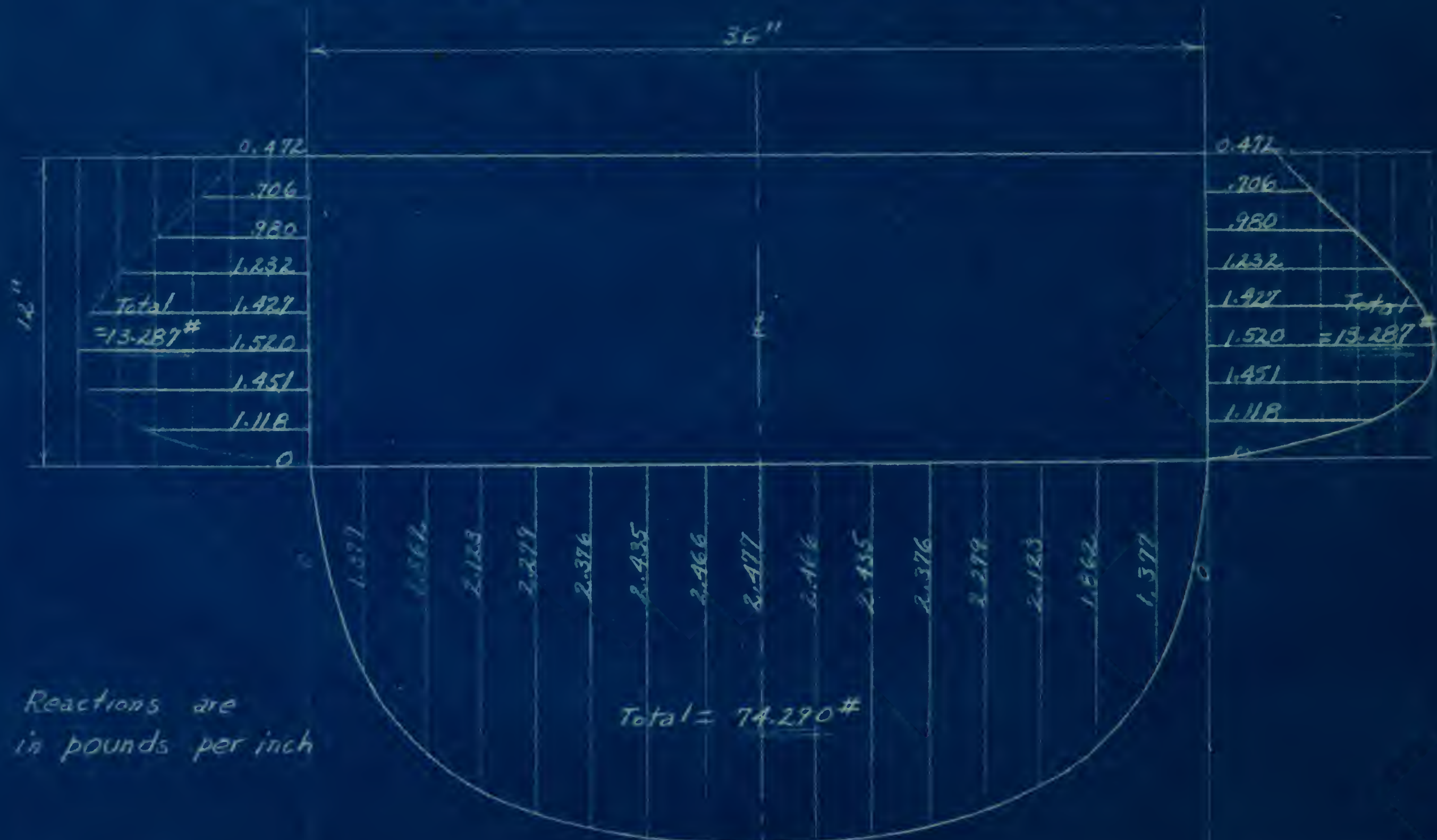
SPRING

SPRING STEEL

FULL SCALE

APPENDIX: THEORETICAL SEAT PRESSURES

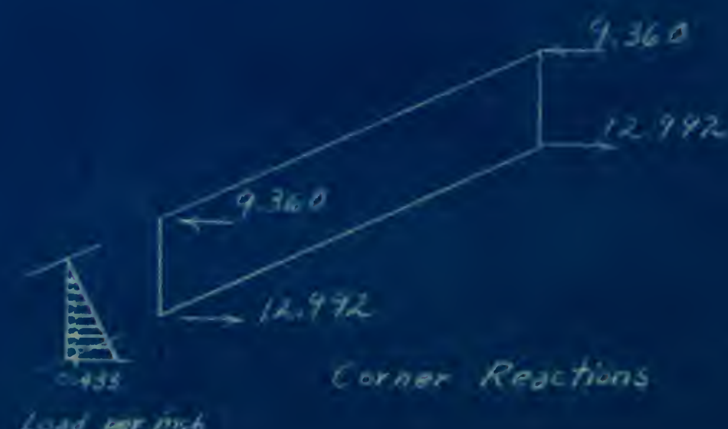
AUG. 23, 1948 BU. OF Y. & D



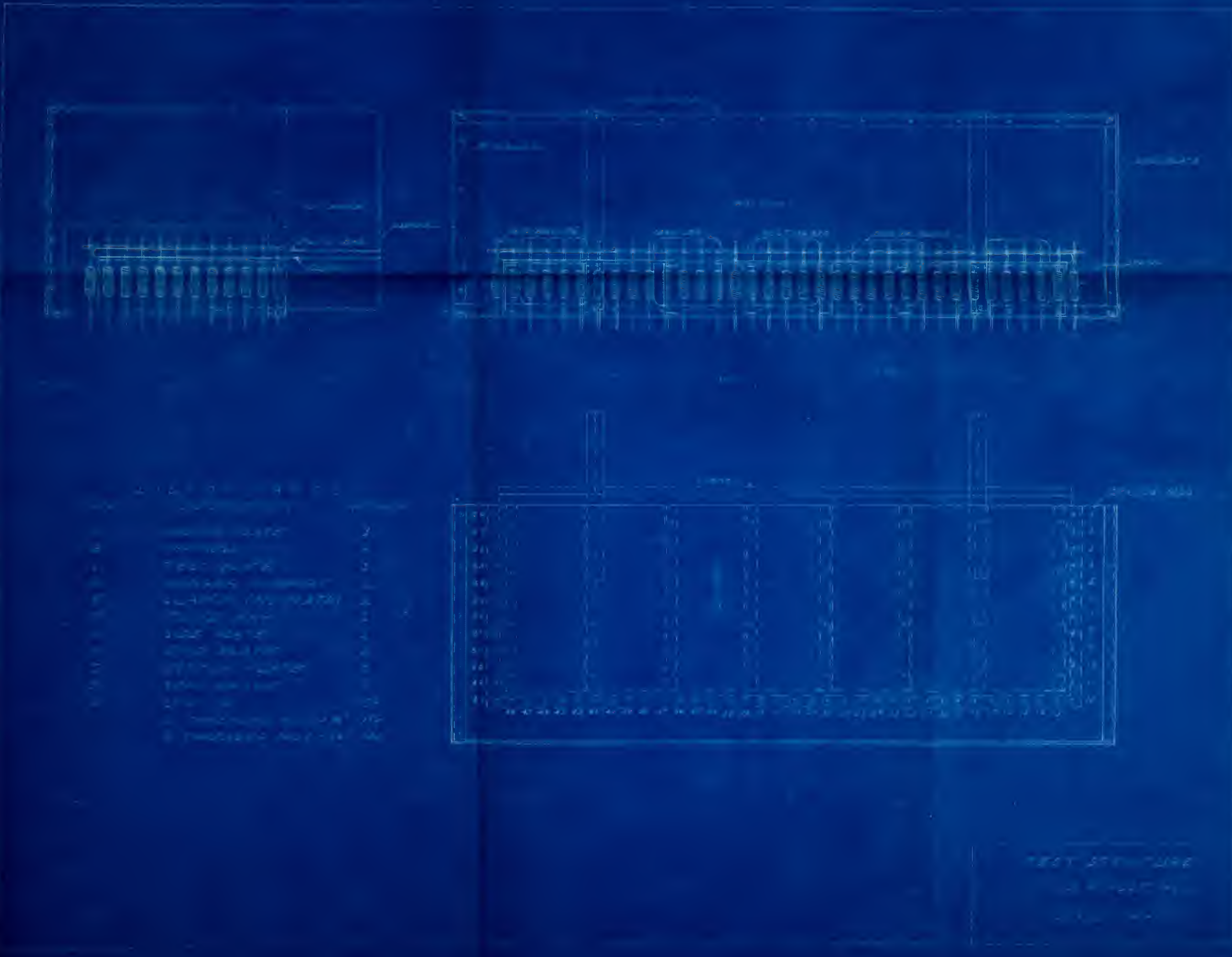
APPENDIX: (CONT'D)

Sum of distributed reactions = 100.864 Pounds
Sum of corner reactions = – 7.264 –
Total = 93.600 Pounds

Total load = $.4333 \times \frac{12}{2} \times 36$ = 93.6 Pounds



Corner Reactions


TEST STRUCTURE

Thesis . 7474
C18 Cahn
Reaction analysis of
dry dock caisson.